Lorada & Kiefer

Summer 2005 Microeconomics Qualifying Exam

There are three sections on this exam:

- In the first section there are three questions; you should work all of them.
- In the second section there are two questions; you should work one of them.
- In the third section there are three questions; you should work two of them.

You have 4 hours and 30 minutes (that is, until 1:30 PM) to finish this test. This gives you about 45 minutes per question.

Good luck.

Section 1.
Answer all of the following three questions.

1. Suppose there are N price-taking (i.e., competitive) consumers, all of whom earn the same income $\$m$, all of whom consume two commodities x and y, and all of whom have the identical utility function $U = x^{\alpha}y^{\beta}$ where α and β are positive.

   Suppose there are F price-taking (i.e., competitive) producers of good x, each having the same cost function $C(x)$ for producing x.

   (a) If $N = 1$ and $F = 1$ but the agents still act competitively, and if $C(x) = x^2$, how will changes in β affect the equilibrium price of x?

   (b) If N and F are arbitrary natural numbers and if the form of $C(x)$ is unspecified (but the firms' second-order conditions are met), how will changes in β affect the equilibrium price of x?

   (c) If $N = 1$ and $F = 1$ and $C(x) = x^2$ but the agents still act competitively, how does the consumer think changes in β will affect the equilibrium price of x?

2. The Second Theorem of Welfare Economics states that (given certain conditions) it is possible to achieve any Pareto Efficient allocation via competitive markets, if, before the competitive markets open, lump-sum transfers can be imposed.

   Suppose there are two consumers in an economy; the first has utility function $U_a = x_a\, y_a$ and the second has utility function $U_b = x_b\, y_b$, where x and y are the two goods and "a" and "b" represent the first and second consumer, respectively. The initial allocation is $(\omega_{xa}, \omega_{ya})$ for the first consumer and $(\omega_{xb}, \omega_{yb})$ for the second consumer. For an arbitrary Pareto Efficient point in this economy, what lump-sum transfers (T_x, T_y) *from* consumer "a" *to* consumer "b" are required in order for that Pareto Efficient point to be the outcome of competitive markets? (It is possible for T_x or T_y to be negative.) Hint: I found it easier to calculate the Pareto Efficient points using the "social weights" approach.

3. Consider a duopoly supplying a pair of similar goods, gadgets and gizmos. Both are produced by combining capital k and labor l. One firm makes gadgets, and the second makes gizmos.

(a) Both firms have identical cost functions,

$$c_g(v, r, y_g) = \min(v, r)\, y_g$$
$$c_z(v, r, y_z) = \min(v, r)\, y_z$$

where v is the capital rental rate and w the wage rate. What is the gadget production function? Is it homogeneous of degree 1? Monotonic? Convex? Plot the isocost and isoquant curves; give an interpretation these curves.

(b) Suppose that $v = 3$ and $w = 2$, and that both markets are competitive. The direct demand functions are

$$\begin{aligned} \text{gadgets} \quad & y_g = 6 - p_g + \tfrac{2}{3} p_z \\ \text{gizmos} \quad & y_z = 6 - p_z + \tfrac{2}{3} p_g \end{aligned}$$

What is the equilibrium? Express your answer as (y_g, y_z, p_g, p_z).

(c) What is the outcome in the case that they come to a differentiated-product-Bertrand equilibrium (and $v = 3$ and $w = 2$)? Plot the reaction curves to illustrate. Why is it a Nash equilibrium?

(d) Suppose that the two firms form a cartel, but continue to market both gadgets and gizmos (and $v = 3$ and $w = 2$). What is the equilibrium?

(e) Assuming that these are the only possible price strategies, fill in the payoff matrix.

profit payoffs: (gadgets, gizmos)		Gizmos	
		cartel	Bertrand
Gadgets	cartel		
	Bertrand		

(f) Suppose that this simultaneous pricing game is repeated infinitely many times. Consider the *trigger* threat:

- play cartel in the first game;
- thereafter play cartel, unless the rival plays Bertrand in the previous game,
- then punish rival by playing Bertrand forever.

What outcome will occur if both players follow this strategy? Under what conditions is (cartel, cartel) the subgame-perfect Nash equilibrium of the infinitely repeated game?

(g) Now consider the *tit-for-tat* threat:

- play cartel in the first game;
- thereafter play whatever the rival did in the previous game.

What outcome will occur if both players follow this strategy? Under what conditions is (cartel, cartel) the subgame-perfect Nash equilibrium of the infinitely repeated game?

(h) Which threat is more likely to result in the cartel outcome? Explain.

Section 2.

Answer one of the following two questions.

1. In this problem, if you would like to use the Envelope Theorem, you need not prove it. On the other hand, if you assert convexity or concavity of a function, you should prove that.
   (a) Derive the basic comparative-statics results for a competitive profit-maximizing firm when all relevant prices change.
   (b) Derive the basic comparative-statics results for a competitive cost-minimizing firm when all relevant prices change.
   (c) Compare and contrast your answers to (a) and (b). Are they contradictory or consistent with each other?

2. (a) Give an example of a function of one variable (not two or more variables) which is both quasiconcave and quasiconvex.
   (b) Give an example of a function of one variable (not two or more variables) which is not quasiconcave but is quasiconvex.
   (c) Give an example of a function of one variable (not two or more variables) which is quasiconcave but not quasiconvex.
   (d) Give an example of a function of one variable (not two or more variables) which is quasiconcave but not concave.
   (e) Give an example of a function of one variable (not two or more variables) which is quasiconvex but not convex.

   You do not have to choose functions which are defined over the entire real line; a domain which is only part of the entire real line is acceptable.

Section 3.

Answer two of the following three questions.

1. "Democracy is imperfect, but better than the alternative."

   Discuss.

2. Imagine a 2 by 1 economy. The single consumer, Robinson, consumes two goods, leisure x_1 and tacos x_2. His preferences are given by

$$U(x_1, x_2) = x_1 + 2\sqrt{x_2}\,;$$

   his endowment is $\omega = (\omega_1, \omega_2) = (2, 0)$.

   Robinson owns a taco firm. Tacos can be produced according to the production function $y_2 = |y_1|$. General equilibrium is described by $x_1 = \omega_1 + y_1$ and $x_2 = y_2$. Define the wage rate (price of leisure) as 1.

   (a) Consider perfect competitive regime in the taco market. What quantities of tacos and leisure are consumed, what is the price of a taco?

   (b) For a monopoly regime, find its equilibrium. What is the monopoly price of a taco? Illustrate your answer.

   (c) Assume that the taco monopoly can impose first-degree price discrimination on the consumer. Find its equilibrium.

   (d) How much would Robinson be willing to pay to shift from the monopoly to competitive regime? Illustrate your answer with a diagram. Comment on the welfare implications of this problem.

3. Barbie and Ken consume a private good, coffee x_i, and a public good, poetry G. The utility functions and endowments are given as follows:

Barbie	$U_b = \min(x_b, G)$,	$\omega_b = 3$,
Ken	$U_k = \min(x_k, G)$,	$\omega_k = 3$.

   Each citizen may make a contribution g_i toward the provision of poetry, but such contributions reduce private consumption according to the budget constraint

$$\omega_i = x_i + g_i\,.$$

   The private good can be transformed into the public one according to the transformation function

$$x_b + x_k + G - \omega_b - \omega_k = 0\,.$$

Finally, Barbie and Ken agree on the Rawlsian social welfare function,

$$W = \min(U_k, U_b).$$

(a) Plot reaction curves in g_k–g_b space. Add indifference curves to your g_k–g_b diagram.

(b) Find the Nash equilibrium.

(c) Is the allocation $(G, x_b, x_k) = (2, 2, 2)$ feasible? Is it Pareto efficient? Given these endowments, find all Pareto efficient allocations. Illustrate your answer in g_k–g_b space.

(d) Show that the Nash Equilibrium is also the Rawlsian social optimum. Illustrate your answer in U_k–U_b space.